Structure of Matter - I
June 23, 2016

PROBLEM 1. On electronic structure [15 ptn]

Consider a tin 1+ ion (Sn^{1+}) in the $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 7d$ excited electronic configuration.

a) Sketch the radial part of the 7d wave function (rR_{7d}). [1 ptn]

b) Calculate using the Bohr approximation the binding energy of Sn^{1+}(..7d). [1 ptn]

c) The real binding energy is 2.2 eV. Determine the effective nuclear charge seen by the 7d electron. [1 ptn]

d) Why is the effective nuclear charge higher than 2+. [2 ptn]

e) Compared to the 7d electron, would a 7s electron experience a higher nuclear charge and why. [1 ptn]

f) The excited 7d electronic configuration has a lifetime of 7.5 ns. Calculate the probability that after 30 ns the ion is still in its .....7d state. [2 ptn]

Now consider a tin 5+ ion in its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^7$.

g) Determine the ground term of Sn^{5+}. [3 ptn]

h) Determine the ground level of Sn^{5+}. [1 ptn]

i) The tin isotope we are considering has a nuclear spin of I=3/2. Due to the nuclear spin the ground level splits up into hyperfine levels. Determine all the possible hyperfine levels. (In case you could not determine the ground level (question 1h) then you may use J=5/2). [1 ptn]

j) What are the answers to questions 1g), 1h), and 1j) when four more electrons are removed from the tin ion producing a Sn^{9+} ion in its ground electronic configuration. [2 ptn]

PROBLEM 2. On nuclear structure [13 ptn]

Consider the scandium nucleus $^{49}_{21}Sc$

a) Calculate the charge density [in units of elementary charge per fm^3] of the nucleus. [1 ptn]

b) Determine the nuclear spin J and the parity of this isotope. Assume the generic sequence of nuclear shell filling: 1s, 1p, 1d, 2s, 1f, 2p, 1g, ... [3 ptn]

c) Determine the most likely lowest excited nuclear level? [2 ptn]

d) Assume that the $^{49}_{21}Sc$ isotope consists fully out of antimatter (antiprotons and antineutrons). Determine the nuclear spin J and the parity of this antimatter nucleus. [2 pnts]

e) Give the reaction formulae for α, β^+, and β^- decay. Label the daughter nucleus as Y. [2 pnts]

f) The binding energies of nuclei are well approximated by:

$$B = aA - bA^{2/3} - \frac{dZ^2}{A^{1/3}} - s\frac{(N-Z)^2}{A} - \frac{\delta}{A^{1/2}}$$

with a = 15.84 MeV, b = 18.33 MeV, d = 0.714 MeV, s = 23.2 MeV, and δ is 11, 0, and -11 MeV for odd-odd, even-odd, and even-even nuclei, respectively. Determine the difference in nuclear binding energy between the parent X and daughter Y after β^- decay. You may assume the constant d to be 0. [2 pnts]

g) Is β^- decay a likely decay channel for $^{49}_{21}Sc$, and why. [1 pnts]

PROBLEM 3. Elementary particles [12 ptn]

Consider a charmed Λ_c baryon with quark content udc which decays into $p + K^- + \pi^+$.

[quark compositions of π^+ : $u\bar{d}$ and K^-: $s\bar{u}$]

a) Determine the charge of this Λ_C baryon? [1 ptn]

b) Determine the hypercharge of this Λ_C baryon? [1 ptn]

c) Verify that the conservation laws for lepton and baryon number are respected? [1 ptn]

d) Which conservation law is violated? [1 ptn]

e) By which force(s) is the decay driven? [1 ptn]

f) What is the approximate time scale of the decay? [1 ptn]

g) Consider the J=3/2 family of charmed baryons. The 6 baryons of this family are: udc, uuc, usc, ddc, dsc, and ssc. Determine the hypercharge Y, the azimuthal isospin I_3, and the isospin of these six baryons. [2 ptn]

h) J=3/2 implies that all spins of the quarks can be equal. Why are baryons with two equivalent quarks (e.g. uuc) allowed to exist and indicate why in the first place the occurrence of such baryons with two equivalent quarks appears to be surprising? [2 ptn]

i) What are the values of the color charges I_3^C and Y^C of the udc baryon and why? [2 ptn]